

Application of a Self-Organizing State Space Model to the Leonid Meteor Storm in 2001

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Abstract

The Leonids show meteor storms in a period of 33 years, and known as one of the most active meteor showers. It has recently shown a meteor stream consisting of several narrow dust trails made by meteoroids ejected from a parent comet. Hence, an analysis of the temporal behavior of the meteor flux is important to study the structure of the trails. However, statistical inference for the count data is not an easy task, because of its Poisson characteristics. We carried out a wide-field video observation of the Leonid meteor storm in 2001. We formulated a state-of-the-art statistical analysis, which is called a self-organizing state space model, to infer the true behavior of the dust density of the trails properly from the meteor count data. From this analysis, we found that the trails have a fairly smooth spatial structure, with small and dense clumps that cause a temporal burst of meteor flux. We also proved that the time behavior (trend) of the fluxes of bright meteors and that of faint meteors are significantly different. In addition we comment on some other application of the self-organizing state-space model in fields related to astronomy and astrophysics.

Key words: comets: individual (comet 55P/Tempel-Tuttle) — interplanetary medium — meteors, meteoroids — methods: statistical

1. Introduction

1.1. Leonid Meteor Storm as a Stochastic Counting Process

The Leonids show meteor storms in a period of 33 years, and known as one of the most active meteor showers. It is believed that the Leonid meteor storm occurs when the Earth approaches the orbit of the dust trails of the comet 55P/Tempel-Tuttle. Many authors have predicted that the Leonid meteor storm would occur in 2001 (see e.g., Watanabe et al. 2002 and references therein). It has been recognized that a meteor stream consists of several narrow

dust trails, each of which is made by meteoroids ejected at a particular return of the parent comet (e.g., Watanabe et al. 1997; Brown et al. 2002). Hence, the temporal behavior of the meteor flux provides important information of the spatial and density structure of the dust trails.

We carried out a wide-field video observation of the Leonid meteor storm in 2001 (Shiki et al. 2003), and found the following results through a statistical analysis:

1. The time variation of the hourly rate (HR) shows many small peaks.
2. The HR profile of bright meteors and that of faint meteors are significantly different.
3. The small peaks are associated with a burst of faint meteor flux.

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4. Some signatures of Poisson-like process are found in the interval distribution of meteors and in their autocorrelation function.

Many previous studies tried to analyze the time variation of the meteor flux (e.g., Watanabe et al. 1997 and references therein). However, it is not a trivial task to infer the true behavior of the trail density, since it suffers from a strong statistical fluctuation. The difficulty is due to the fact that measuring the meteor flux is a kind of typical counting process, whose fluctuation is basically not Gaussian, but is characterized by a Poisson distribution. The Poisson counting process has a variance equal to its mean value, i.e., when the mean is large, the associating variance is also large. This character prevents us from applying the popular classical time series analysis methods to the count data, because some basic assumptions are utterly violated (e.g., Brockwell, Davis 1996; Cameron, Trivedi 1998).

1.2. Development of Time Series Analysis

Classical time series analyses have often appeared in an astronomical context. Data from astronomy as well as from other physical, biological, or econometric studies consist of a sequence of numbers, $\{x_1, x_2, x_3, \dots, x_T\}$, obtained by measuring the quantity x_t during a sequence of times. The subscripts represent discrete time that runs from $t = 1$ to T . This discrete treatment of time is sufficient for a variety of practical applications.¹ A comprehensive summary of the classical time series analysis and its applications to astronomical datasets can be found in Scargle (1981).

Today, in order to handle a wider range of time series, including some latent variables, the *state space model* has been proposed from the field of engineering and system optimal control. Especially, the merit of using the state space form is that it can properly treat time-varying parameters in the system. We note that classical time series models can be defined as special cases of the general state space model. The well-known Kalman filter (Kalman 1960) gives an algorithm to estimate the system parameters recursively, i.e., it gives one-step-ahead estimations (called ‘filtering’) everytime we have a new data point in the series.

A Kalman filter assumes the Gaussianity for the system noise terms, which is not suitable for the data we often encounter in astronomical applications. Further, as seen in section 3.1, the Kalman filter is designed for a linear system. A generalized state space model with nonlinearity and non-Gaussianity has been proposed to overcome these shortcomings (Kitagawa, Gersch 1996, and references therein). An excellent guide for their applications can be found in Brockwell, Davis (1996).

However, there still remains an additional difficulty concerning astronomical data analysis. Astronomers frequently meet a situation in which they must handle a

dataset with small counts, such as a very low-level signal of the faintest sources. Count data introduce complications of discreteness and heteroskedasticity.² The inclusion of zero counts appears to be a pitfall to apply, e.g., usual regression methods. Despite its frequency that we should tackle such datasets, time series models for counting data are in their infancy, yet remarkably many models have been developed. Cameron, Trivedi (1998) provides a thorough discussion on general counting problems, including time series counting data.

Now we close the chronicle of time series analysis. The development is concisely summarized in Kitagawa, Sato (2001). In this paper, we propose a suitable method of analyzing time series count data, which sometimes have zeros in the sequence. This approach, the self-organizing state space model, has been developed only very recently (Kitagawa 1998; Higuchi 1999, and references therein). It makes extensive use of a large computing power of modern computers.

In this paper we statistically formulate the count data of the Leonid meteor storm. The count rate obeys a Poisson process with a latent, time-varying Poisson intensity, λ_t . We made an attempt to estimate the temporal behavior of the hidden parameter λ_t from the observed count data. We should note that our data have some gaps in observations caused by the length of video tapes and other reasons. Our method can easily overcome such gaps, and properly infer the value in the observational time gaps.

The rest of the present paper is as follows. In section 3, we formulate a self-organizing state space model of the Leonid count data. We start with a linear state space model, and then develop toward more general methods. We present the results and discussions in section 4. section 5 is devoted to a summary.

2. Data

The original data consist of the magnitudes and observed time of the meteors, which were recorded on a video tape. Although they include some sporadic meteors and meteors belonging to other meteor showers, here we concentrate only on the Leonids in the analysis. Detailed data descriptions are found in Shiki et al. (2003).

In the usual manner of the meteor-shower analysis, an hourly rate (HR) is used to describe the time variation of the meteor flux. However, in this study, it is more appropriate to present the data based on the count rate per minute; hence, we use the count rate [min^{-1}] throughout this paper.

3. Method

3.1. State Space Model

A classical state space model consists of the following equations:

$$x_t = F_t x_{t-1} + G_t v_t, \quad v_t \sim N(0, Q_t), \quad (1)$$

$$y_t = H_t x_t + w_t, \quad w_t \sim N(0, R_t). \quad (2)$$

² This technical term means that the variance is not constant.

¹ Here, we concentrate on a set of equally sampled time series data, but, as mentioned later, the method presented in this article can also be applied to irregularly sampled data.

Here, x_t is called a state vector, which represents the (unobserved) state of the system, and y_t is the observed sequence of data. The idea underlying the model is that the development of the system over time is determined by x_t according to equation (1). However, because x_t cannot be observed directly, we must base the analysis on observations, y_t . We call equation (1) the state equation, and equation (2) the observation equation. The error terms, v_t and w_t , are distributed according to Gaussian probability distribution functions, $N(0, Q_t)$ and $N(0, R_t)$, respectively, where Q_t and R_t are covariance matrices.

Matrices F_t , G_t , H_t , Q_t , and R_t are initially assumed to be known, and the error terms, v_t and w_t , are assumed to be serially independent and independent of each other at all time. In practice, some or all of the matrices depend on the elements of an unknown parameter vector, θ .

A considerable advantage of the state space approach is the ease with which missing observations can be dealt with. The estimation problems in time series analysis can be classified into the following three categories with respect to the dependence on the observed data $\{y_1, \dots, y_t\}$ to estimate the state vector x_t :

1. $y_{1:t-1} \equiv \{y_1, \dots, y_{t-1}\}$: prediction,
2. $y_{1:t} \equiv \{y_1, \dots, y_t\}$: filtering, and
3. $y_{1:u} \equiv \{y_1, \dots, y_u\}$ ($u > t$) : smoothing.

Kalman recursion equations give a one-step-ahead prediction of the state vector, x_t , and its error covariance matrix, by using $\{y_1, \dots, y_{t-1}\}$, and filter the series when we have new data, y_t . We do not go any further into the details of the Kalman filter here. For implementation, see, e.g., Harvey (1981), Brockwell, Davis (1996) and Durbin, Koopman (2001). If we have a missing in data sequence, we simply perform prediction without filtering, and go to the next step. This point is extensively discussed by Akaike, Kitagawa (1999).

3.2. Generalized State Space Model

We can consider a nonlinear non-Gaussian state space model as being an extension of the linear case (Kitagawa 1987):

$$x_t = F_t(x_{t-1}, v_t), \quad (3)$$

$$y_t = H_t(x_t, w_t). \quad (4)$$

Again, the first is the state equation and the second is the observation equation. These times, v_t and w_t , are the system and observation noise with non-Gaussian densities, $q_t(v_t)$ and $r_t(w_t)$, respectively. The initial state x_0 is assumed to be distributed with the probability density $p_0(x_0)$. Functions $F_t(x, v)$ and $H_t(x, w)$ are nonlinear ones of the state vector and noise (Brockwell, Davis 1996; Durbin, Koopman 2001; Kitagawa, Sato 2001).

It is convenient to express the model in a general form based on the conditional distributions:

$$x_t = Q_t(\cdot | x_{t-1}), \quad (5)$$

$$y_t = R_t(\cdot | x_t). \quad (6)$$

With this general state space model, we can handle discrete-valued time series as well as discrete-state models.

For general state space models, the conditional distributions become non-Gaussian and their distributions cannot be completely specified by the mean vectors and the covariance matrices, which is different from the case of a Gaussian linear state space model and a Kalman filter. A non-Gaussian filter is expressed as follows:

$$p(x_t | y_{1:t-1}) = \int p(x_t | x_{t-1}) p(x_{t-1} | y_{1:t-1}) dx_{t-1}, \quad (7)$$

$$p(x_t | y_{1:t}) = \frac{p(y_t | x_t) p(x_t | y_{1:t-1})}{p(y_t | y_{1:t-1})}, \quad (8)$$

where $p(y_t | y_{1:t-1})$ is the predictive distribution of y_t ,

$$p(y_t | y_{1:t-1}) = \int p(y_t | x_t) p(x_t | y_{1:t-1}) dx_t. \quad (9)$$

However, a direct implementation of the formula requires computationally intense numerical integration that can only be feasible for some limited case.

3.3. Monte Carlo Filter

Instead of approximating the distribution and performing heavy numerical integration, we can use Monte Carlo filtering, i.e., producing a large number of realizations which can be extracted from the distribution (Kitagawa 1996). This method needs much less computational power.

The procedure is as follows:

1. Generate a random number, $x_0^{(j)} \sim p_0(x_0)$, for $j = 1, \dots, N$.
2. Repeat the following steps for $t = 1, \dots, T$:
 - (a) Generate a random number $v_t^{(j)} \sim q(v)$ for $j = 1, \dots, N$.
 - (b) Compute $p_t^{(j)} = F[x_{t-1}^{(j)}, v_t^{(j)}]$ for $j = 1, \dots, N$.
 - (c) Compute $w_t^{(j)} = p[y_t | p_t^{(j)}]$ for $j = 1, \dots, N$.
 - (d) Generate $x_t^{(j)}$ ($j = 1, \dots, N$) by resampling of $p_t^{(j)}$ ($j = 1, \dots, N$).

In step 2(b), $p_t^{(j)} = F[x_{t-1}^{(j)}, v_t^{(j)}]$ can be considered to be independent realizations from the predictive distribution, $p(x_t | y_{1:t-1})$. Given the observation y_t and the realization $p_t^{(j)}$, we obtain the importance weight, $w_t^{(j)}$ [step 2(c)], i.e., the likelihood with respect to the observation, y_t . The posterior probability of the realization is

$$\begin{aligned} & \text{Prob}[x_t = p_t^{(j)} | y_{1:t}] \\ &= \text{Prob}[x_t = p_t^{(j)} | y_{1:t-1}, y_t] \\ &= \frac{p[y_t | p_t^{(j)}] \text{Prob}[x_t = p_t^{(j)} | y_{1:t-1}]}{\sum_{k=1}^N p[y_t | p_t^{(k)}] \text{Prob}[x_t = p_t^{(k)} | y_{1:t-1}]} \\ &= \frac{w_t^{(j)}}{\sum_{k=1}^N w_t^{(k)}}. \end{aligned} \quad (10)$$

This means that the cumulative distribution function of $\text{Prob}[x_t = p_t^{(j)} | y_{1:t}]$ can be expressed by a step function,

$$\text{Prob}[x_t = p_t^{(j)} | y_{1:t}] = \frac{1}{\sum_{k=1}^N w_t^{(k)}} \sum_{j=1}^N w_t^{(k)} I[x, p_t^{(j)}], \quad (11)$$

which has jumps at p_t^1, \dots, p_t^N with step sizes $w_t^{(1)}, \dots, w_t^{(N)}$. Here, $I(x, z)$ is the Heaviside function, which has a unit jump at $x = z$.

For the next step of the prediction, we must represent this distribution function by an empirical distribution with the form

$$\text{Prob}[x_t = p_t^{(j)} | y_{1:t}] = \frac{1}{N} \sum_{j=1}^N I[x, p_t^{(j)}]. \quad (12)$$

This can be done by resampling of $\{p_t^{(1)}, \dots, p_t^{(N)}\}$ with probabilities,

$$\text{Prob}[x_t = p_t^{(j)} | y_{1:t}] = \frac{w_t^{(j)}}{\sum_{k=1}^N w_t^{(k)}}. \quad (13)$$

We calculate this step using the von Neumann's acceptance-rejection method (see e.g., Knuth 1998 for details).

One of the main purposes of the time series analysis is to estimate some hidden parameters, θ , from the observed data. The likelihood of the time series model specified by the parameter θ is obtained by

$$L(\theta) = p(y_1, \dots, y_T) = \prod_{t=1}^T p(y_t | y_{1:t-1}; \theta). \quad (14)$$

Along with the above discussion, we can approximate the conditional density by

$$p(y_t | y_{1:t-1}, \theta) \simeq \frac{1}{N} \sum_{j=1}^N w_t^{(j)}. \quad (15)$$

3.4. Self-Organizing State Space Model

In principle, the maximum likelihood estimate of θ is obtained by maximizing the log-likelihood, $\log L(\theta)$. However, in practice the sampling error and long computational time often renders the direct maximum likelihood method impractical. To remedy this problem, Kitagawa (1998) proposed a sophisticated method. Instead of estimating the parameter θ by the maximum likelihood, we consider a Bayesian estimation by augmenting the state vector, x_t , with an unknown parameter, θ , as

$$z_t = \begin{pmatrix} x_t \\ \theta_t \end{pmatrix}, \quad (16)$$

and construct the state space model for z_t as

$$z_t = F^*(z_{t-1}, v_t), \quad (17)$$

$$y_t \sim R(\cdot | z_t). \quad (18)$$

This is called ‘the self-organizing state space model’. In equation (16), the parameter is unknown, but constant such that $\theta_t = \theta_{t-1} = \dots = \theta$. This can immediately be extended to the time-varying (hyper)parameter case, but we do not go any further here.

3.5. Model for the Leonid Meteor Storm

Here, we formulate our problem to estimate the temporal trend of the Leonid meteor shower hidden by a statistical fluctuation, which may be Poissonian. For the Poisson count process, the observation equation is expressed as

$$y_t \sim \text{Poisson}(\lambda_t) = \frac{\lambda_t^{y_t} e^{-\lambda_t}}{y_t!}, \quad t = 1, \dots, T, \quad (19)$$

(Higuchi 1999; Higuchi 2001; Kitagawa, Sato 2001). The system model becomes quite simple, as

$$x_t = \begin{pmatrix} \mu_t \\ \log \sigma_{\mu,t}^2 \end{pmatrix} = \begin{pmatrix} \mu_{t-1} \\ \log \sigma_{\mu,t-1}^2 \end{pmatrix} + \begin{pmatrix} v_t \\ 0 \end{pmatrix}, \quad (20)$$

where $\mu_t \equiv \log \lambda_t$. We adopt a first-order smooth trend such that $\mu_t = \mu_{t-1} + v_t$, $v_t \sim N(0, \sigma_v^2)$. This treatment enables us to handle an arbitrary trend, because it only assumes that the first-order difference of the trend is small. The parameter σ_μ^2 ($\log \sigma_\mu^2$) is simultaneously estimated by a recursive procedure.³ As recommended by Higuchi (2001), we set $\log \sigma_\mu^2 \sim U([-6.0, -2.0])$ as the initial distribution of σ_μ^2 , where $U([a, b])$ denotes the uniform distribution between a and b .

As already mentioned above, Leonid meteors seem to behave like a Poisson process locally, and hence this model is appropriate to describe this data (see Shiki et al. 2003).

4. Results and Discussions

The observations were made from 14^h41^m to 20^h03^m UT (322 minutes), hence $T = 322$ in equation (19).

4.1. Global Behavior

We present the count rate data of the Leonid meteor storm and its estimate for the true density distribution in Figure 1. At a glance we can see that the density estimate has a fairly smooth spatial structure, and that the violent statistical fluctuation is significantly suppressed by the self-organizing state space method. The estimation uncertainty, which is simultaneously estimated by the recursive estimation process, is ~ 0.1 . The $\pm 1\text{-}\sigma$ uncertainty envelopes are also plotted in Figure 1, but it is hard to resolve on the figure. This result shows that most of the ‘spikes’ in the meteor count data are merely a consequence of the Poisson fluctuation, and have no physical substance. Therefore, even if a spike appears to be strong, it may be explained by a large variance (standard deviation) of the Poisson process.

By our method, only those spikes which cannot be regarded as a mere fluctuation are detected in the trend estimate. The most prominent feature is the burst of meteor flux just before the first global peak of the storm. We can observe some other small peaks at 100, 130, 160, 210, 250, 260, and 270 min in Figure 1. They clearly correspond to the peaks suggested by an analysis of Shiki et al. (2003). Interestingly, the first spike of the count at

³ The logarithm is defined as $\log x \equiv \log_{10} x$. It is set to conserve the positive definiteness of the trend.

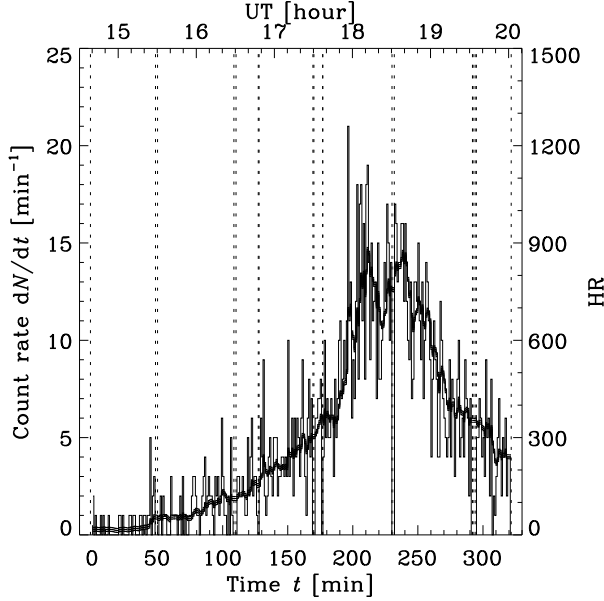


Fig. 1. Count rate data of the Leonid meteor storm and its estimate for the true density distribution. The vertical dotted lines indicate the boundaries of the data. Data are missing in small intervals between the dotted lines. The observed counts are shown by the thin histogram, and the thick zigged curve presents an estimate of the true count behavior, λ_t . On the right-hand side we show the HR (hourly rate) for the convenience of readers.

45 min ($\sim 15^{\text{h}}25^{\text{m}}$) appeared to be a sudden increase of the underlying meteor flux, and not a spiky burst of count in the estimate.

Thus, we conclude that the true temporal trend of the dust trail which caused the Leonid meteor storm is globally smooth, with small and dense clumps associated with bursts of meteor counts. This confirms the suggestion from a classical analysis by Shiki et al. (2003), and provides a statistically rigorous basis on it.

4.2. Magnitude Dependence

Next, we divided the data into two classes, bright and faint samples, and applied the self-organizing state space method to both of them, just as we did for the whole sample. We set the boundary of the bright and faint samples at 3 mag. The results of the bright and faint samples are shown in Figures 2 and 3, respectively.

A drastic difference is found between the counts of the bright and faint samples. The most striking feature is that there is no trend of bursts in bright sample counts at 200 min ($\sim 6^{\text{h}}00^{\text{m}}$ UT), whereas a prominent burst exists in the faint counts. Other weaker bursts also stem from the faint count behavior. In other words, the bright sample count is relatively smooth and its variation is small, while the faint count temporarily varies with a very similar trend of the total count profile. This clearly shows that most of the bursts have been dominated by meteors fainter than 3 mag.

We also find a clear excess of bright meteors to faint ones after 270 min ($19^{\text{h}}20^{\text{m}}$ UT). It is an unexpected

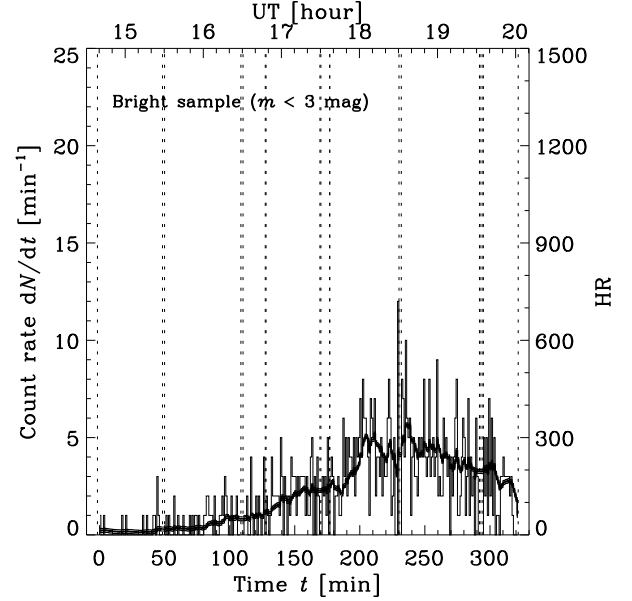


Fig. 2. Count rate data of the Leonid meteor storm for meteors brighter than 3 mag, and its estimate for the true density distribution. Again, the vertical dotted lines indicate the boundaries of the data. The observed counts are shown by the thin histogram, and the thick zigged curve presents an estimate of the true count.

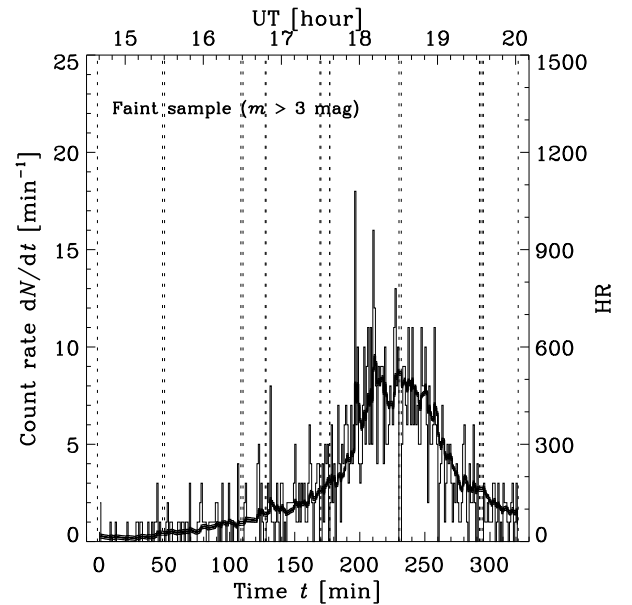


Fig. 3. Same as Figure 2, except that it is for meteors fainter than 3 mag.

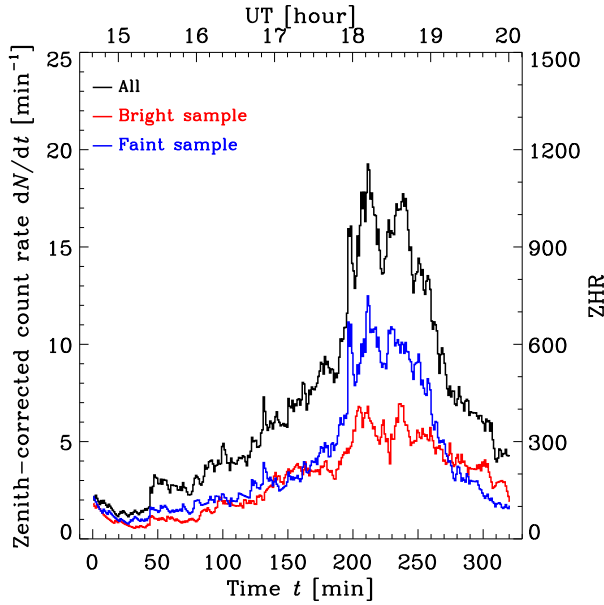


Fig. 4. Zenith-corrected meteor flux estimates.

trend, because it is widely accepted that fainter meteors are more numerous than brighter ones. One may suspect the effect of the elevation of the radiation point, but this result is unchanged by the correction because the correction has no magnitude dependence. This suggests a bias in the distributions of larger and smaller meteorites, which may be the origins of brighter and fainter meteors, respectively.

Hughes (1973) reported a clear difference in the cumulative influxes of meteors brighter and fainter than 3 mag. Our result may be closely related to his conclusion, but we do not go further here.

So far, we have not corrected the zenith effect on the meteor flux so as to avoid any unnecessary intricacy in statistical modeling. If one may wish to have a zenith-corrected count rate (or equivalently, ZHR), we should merely make a correction of the obtained estimate here. Figure 4 shows the corrected meteor flux and ZHR, together with those of bright and faint subsamples. For the correction, we simply multiplied $1/\sin h$ (h : elevation of the radiation point) to the flux estimates. If we use an empirical formula of the form $1/(\sin h)^\gamma$ ($\gamma \simeq 1.4$: see e.g., Jenniskens 1994), the trend around 15^h–17^h would be more emphasized. We note that the sum of the estimates for these subsamples perfectly agrees with the estimates for a whole sample. This also shows that our approach is very powerful, robust, and consistent for this type of analysis. In figure 4, the coincidence of the burst spikes in the whole sample and faint subsample is impressive.

4.3. Future Prospects of the Self-Organizing State Space Model for Astrophysical Applications

Before closing this article we would like to devote a subsection to some future prospects for applying the self-organizing state space model approach. This approach

has a very wide range of its applicability: for example, it can be used to estimate an extremely faint optical source variability, and to analyze count rates of low-level photons, X-ray or cosmic ray detectors, which are regarded as representatives of typical count processes.

Another important aspect is its robustness against irregular sampling. Hence, we can easily apply this method to the photometric sequence data of gravitationally lensed objects to measure their time delay in variability.

Higuchi (2001) illustrated an interesting application to an estimation of spiral density wave in Saturn’s ring observed by Voyager, which is known to have a varying frequency along with a radial position (Horn et al. 1996). He applied the self-organizing state space model approach to the data and showed a beautiful result. Sunspot number data are also very popular count process in statistical science (Higuchi 1999).

Thus, we expect a variety of applications of this analysis. Now, with this approach, we do not have to worry about unrealistic assumptions of stationarity nor Gaussianity that are hardly expected for real datasets, in spite of the fact that they are often required for the popular classical time series analysis. Also, we will never be annoyed by irregularly appearing observational gaps, sampling inhomogeneity, or heteroskedasticity that are often inherent in various astronomical datasets.

5. Summary and Conclusions

The Leonids show meteor storms in a period of 33 years, and known as one of the most active meteor showers. It has recently shown a meteor stream consisting of several narrow dust trails made by meteoroids ejected from a parent comet; hence, an analysis of the temporal behavior of the meteor flux is important for studying the structure of the trails. However, statistical inference for the count data is not an easy task, because of its Poisson characteristics. We carried out a wide-field video observation of the Leonid meteor storm in 2001 (Shiki et al. 2003).

In this study, we formulated a state-of-the-art statistical analysis, which is called the self-organizing state space model, to infer the true behavior of the dust density of the trails properly from the meteor count data. From this analysis we found that the trails have a fairly smooth spatial structure, with small and dense clumps, which cause a temporal burst of meteor flux. We also confirmed that the time behavior (trend) of the fluxes of bright and faint meteors are significantly different.

In summary, for the first time we obtained a reliable estimate of the true dust trail density profile by the self-organizing state space approach.

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opment of the self-organizing state space model. T.T.T. has been financially supported by the Japan Society of the Promotion of Science.

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